

3.1.1 EXERCISES

For a link to all of the additional resources available for this section, click [OSTTS Chapter 3 materials](#).

In Exercises 1 - 10, find the degree, the leading term, the leading coefficient, the constant term and the end behavior of the given polynomial.

For help with these exercises, click one or more of the resources below:

- [Identifying the degree, leading term, leading coefficient, and constant term of a polynomial function](#)
- [Identifying end behavior of the graph of a polynomial function](#)

1. $f(x) = 4 - x - 3x^2$

2. $g(x) = 3x^5 - 2x^2 + x + 1$

3. $q(r) = 1 - 16r^4$

4. $Z(b) = 42b - b^3$

5. $f(x) = \sqrt{3}x^{17} + 22.5x^{10} - \pi x^7 + \frac{1}{3}$

6. $s(t) = -4.9t^2 + v_0t + s_0$

7. $P(x) = (x - 1)(x - 2)(x - 3)(x - 4)$

8. $p(t) = -t^2(3 - 5t)(t^2 + t + 4)$

9. $f(x) = -2x^3(x + 1)(x + 2)^2$

10. $G(t) = 4(t - 2)^2(t + \frac{1}{2})$

In Exercises 11 - 20, find the real zeros of the given polynomial and their corresponding multiplicities. Use this information along with a sign chart to provide a rough sketch of the graph of the polynomial. Compare your answer with the result from a graphing utility.

For help with these exercises, click on the resource below:

- [Graphing a polynomial function given in factored form](#)

11. $a(x) = x(x + 2)^2$

12. $g(x) = x(x + 2)^3$

13. $f(x) = -2(x - 2)^2(x + 1)$

14. $g(x) = (2x + 1)^2(x - 3)$

15. $F(x) = x^3(x + 2)^2$

16. $P(x) = (x - 1)(x - 2)(x - 3)(x - 4)$

17. $Q(x) = (x + 5)^2(x - 3)^4$

18. $h(x) = x^2(x - 2)^2(x + 2)^2$

19. $H(t) = (3 - t)(t^2 + 1)$

20. $Z(b) = b(42 - b^2)$

In Exercises 21 - 26, given the pair of functions f and g , sketch the graph of $y = g(x)$ by starting with the graph of $y = f(x)$ and using transformations. Track at least three points of your choice through the transformations. State the domain and range of g .

For help with these exercises, click on the resource below:

- [Given the graph of one function, graph a related function using transformations](#)

21. $f(x) = x^3$, $g(x) = (x + 2)^3 + 1$

22. $f(x) = x^4$, $g(x) = (x + 2)^4 + 1$

23. $f(x) = x^4$, $g(x) = 2 - 3(x - 1)^4$

24. $f(x) = x^5$, $g(x) = -x^5 - 3$

25. $f(x) = x^5$, $g(x) = (x + 1)^5 + 10$

26. $f(x) = x^6$, $g(x) = 8 - x^6$

27. Use the Intermediate Value Theorem to prove that $f(x) = x^3 - 9x + 5$ has a real zero in each of the following intervals: $[-4, -3]$, $[0, 1]$ and $[2, 3]$. For help with this problem, click on [Understanding the Intermediate Value Theorem](#).

28. Rework Example 3.1.3 assuming the box is to be made from an 8.5 inch by 11 inch sheet of paper. Using scissors and tape, construct the box. Are you surprised?¹⁶

In Exercises 29 - 31, suppose the revenue R , in *thousands* of dollars, from producing and selling x *hundred* LCD TVs is given by $R(x) = -5x^3 + 35x^2 + 155x$ for $0 \leq x \leq 10.07$.

29. Use a graphing utility to graph $y = R(x)$ and determine the number of TVs which should be sold to maximize revenue. What is the maximum revenue?

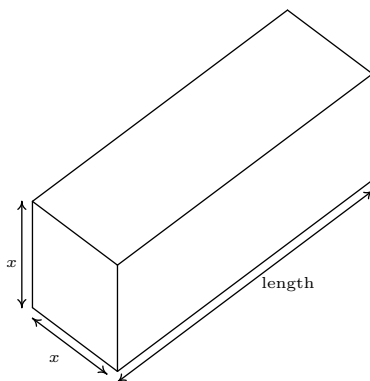
30. Assume that the cost, in *thousands* of dollars, to produce x *hundred* LCD TVs is given by $C(x) = 200x + 25$ for $x \geq 0$. Find and simplify an expression for the profit function $P(x)$. (Remember: Profit = Revenue - Cost.)

31. Use a graphing utility to graph $y = P(x)$ and determine the number of TVs which should be sold to maximize profit. What is the maximum profit?

32. While developing their newest game, Sasquatch Attack!, the makers of the PortaBoy (from Example 2.1.5) revised their cost function and now use $C(x) = .03x^3 - 4.5x^2 + 225x + 250$, for $x \geq 0$. As before, $C(x)$ is the cost to make x PortaBoy Game Systems. Market research indicates that the demand function $p(x) = -1.5x + 250$ remains unchanged. Use a graphing utility to find the production level x that maximizes the *profit* made by producing and selling x PortaBoy game systems.

¹⁶Consider decorating the box and presenting it to your instructor. If done well enough, maybe your instructor will issue you some bonus points. Or maybe not.

33. According to US Postal regulations, a rectangular shipping box must satisfy the inequality “Length + Girth ≤ 130 inches” for Parcel Post and “Length + Girth ≤ 108 inches” for other services.¹⁷ Let’s assume we have a closed rectangular box with a square face of side length x as drawn below. The length is the longest side and is clearly labeled. The girth is the distance around the box in the other two dimensions so in our case it is the sum of the four sides of the square, $4x$.
- Assuming that we’ll be mailing a box via Parcel Post where Length + Girth = 130 inches, express the length of the box in terms of x and then express the volume V of the box in terms of x .
 - Find the dimensions of the box of maximum volume that can be shipped via Parcel Post.
 - Repeat parts 33a and 33b if the box is shipped using “other services”.



34. We now revisit the data set from Exercise 6b in Section 2.5. In that exercise, you were given a chart of the number of hours of daylight they get on the 21st of each month in Fairbanks, Alaska based on the 2009 sunrise and sunset data found on the [U.S. Naval Observatory](#) website. We let $x = 1$ represent January 21, 2009, $x = 2$ represent February 21, 2009, and so on. The chart is given again for reference.

Month Number	1	2	3	4	5	6	7	8	9	10	11	12
Hours of Daylight	5.8	9.3	12.4	15.9	19.4	21.8	19.4	15.6	12.4	9.1	5.6	3.3

Find cubic (third degree) and quartic (fourth degree) polynomials which model this data and comment on the goodness of fit for each. What can we say about using either model to make predictions about the year 2020? (Hint: Think about the end behavior of polynomials.) Use the models to see how many hours of daylight they got on your birthday and then check the website to see how accurate the models are. Knowing that Sasquatch are largely nocturnal, what days of the year according to your models are going to allow for at least 14 hours of darkness for field research on the elusive creatures?

¹⁷See [here](#) for details.

35. An electric circuit is built with a variable resistor installed. For each of the following resistance values (measured in kilo-ohms, $k\Omega$), the corresponding power to the load (measured in milliwatts, mW) is given in the table below. ¹⁸

Resistance: ($k\Omega$)	1.012	2.199	3.275	4.676	6.805	9.975
Power: (mW)	1.063	1.496	1.610	1.613	1.505	1.314

- Make a scatter diagram of the data using the Resistance as the independent variable and Power as the dependent variable.
 - Use your calculator to find quadratic (2nd degree), cubic (3rd degree) and quartic (4th degree) regression models for the data and judge the reasonableness of each.
 - For each of the models found above, find the predicted maximum power that can be delivered to the load. What is the corresponding resistance value?
 - Discuss with your classmates the limitations of these models - in particular, discuss the end behavior of each.
36. Show that the end behavior of a linear function $f(x) = mx + b$ is as it should be according to the results we've established in the section for polynomials of odd degree.¹⁹ (That is, show that the graph of a linear function is "up on one side and down on the other" just like the graph of $y = a_n x^n$ for odd numbers n .)
37. There is one subtlety about the role of multiplicity that we need to discuss further; specifically we need to see 'how' the graph crosses the x -axis at a zero of odd multiplicity. In the section, we deliberately excluded the function $f(x) = x$ from the discussion of the end behavior of $f(x) = x^n$ for odd numbers n and we said at the time that it was due to the fact that $f(x) = x$ didn't fit the pattern we were trying to establish. You just showed in the previous exercise that the end behavior of a linear function behaves like every other polynomial of odd degree, so what doesn't $f(x) = x$ do that $g(x) = x^3$ does? It's the 'flattening' for values of x near zero. It is this local behavior that will distinguish between a zero of multiplicity 1 and one of higher odd multiplicity. Look again closely at the graphs of $a(x) = x(x+2)^2$ and $F(x) = x^3(x+2)^2$ from Exercise 3.1.1. Discuss with your classmates how the graphs are fundamentally different at the origin. It might help to use a graphing calculator to zoom in on the origin to see the different crossing behavior. Also compare the behavior of $a(x) = x(x+2)^2$ to that of $g(x) = x(x+2)^3$ near the point $(-2, 0)$. What do you predict will happen at the zeros of $f(x) = (x-1)(x-2)^2(x-3)^3(x-4)^4(x-5)^5$?

¹⁸The authors wish to thank Don Anthan and Ken White of Lakeland Community College for devising this problem and generating the accompanying data set.

¹⁹Remember, to be a linear function, $m \neq 0$.

38. Here are a few other questions for you to discuss with your classmates.
- (a) How many local extrema could a polynomial of degree n have? How few local extrema can it have?
 - (b) Could a polynomial have two local maxima but no local minima?
 - (c) If a polynomial has two local maxima and two local minima, can it be of odd degree? Can it be of even degree?
 - (d) Can a polynomial have local extrema without having any real zeros?
 - (e) Why must every polynomial of odd degree have at least one real zero?
 - (f) Can a polynomial have two distinct real zeros and no local extrema?
 - (g) Can an x -intercept yield a local extrema? Can it yield an absolute extrema?
 - (h) If the y -intercept yields an absolute minimum, what can we say about the degree of the polynomial and the sign of the leading coefficient?

Checkpoint Quiz 3.1

1. Let $p(x) = -2(x + 3)(2x - 1)^2$.
 - (a) Find the y -intercept of the graph of $y = p(x)$.
 - (b) Determine the end behavior of the graph of $y = p(x)$.
 - (c) Determine the zeros of p and their multiplicities.
 - (d) Graph $y = p(x)$.
2. Find a polynomial function p which satisfies the following criteria:
 - as $x \rightarrow -\infty$, $p(x) \rightarrow \infty$
 - The graph of $y = p(x)$ touches the x -axis at $(-1, 0)$
 - $p(0) = 3$
 - $p(2) = 0$
 - as $x \rightarrow \infty$, $p(x) \rightarrow -\infty$

For worked out solutions to this quiz, click the links below:

- [Quiz Solution Part 1](#)
- [Quiz Solution Part 2](#)

3.1.2 ANSWERS

1. $f(x) = 4 - x - 3x^2$
 Degree 2
 Leading term $-3x^2$
 Leading coefficient -3
 Constant term 4
 As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$
 As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$
2. $g(x) = 3x^5 - 2x^2 + x + 1$
 Degree 5
 Leading term $3x^5$
 Leading coefficient 3
 Constant term 1
 As $x \rightarrow -\infty$, $g(x) \rightarrow -\infty$
 As $x \rightarrow \infty$, $g(x) \rightarrow \infty$
3. $q(r) = 1 - 16r^4$
 Degree 4
 Leading term $-16r^4$
 Leading coefficient -16
 Constant term 1
 As $r \rightarrow -\infty$, $q(r) \rightarrow -\infty$
 As $r \rightarrow \infty$, $q(r) \rightarrow -\infty$
4. $Z(b) = 42b - b^3$
 Degree 3
 Leading term $-b^3$
 Leading coefficient -1
 Constant term 0
 As $b \rightarrow -\infty$, $Z(b) \rightarrow \infty$
 As $b \rightarrow \infty$, $Z(b) \rightarrow -\infty$
5. $f(x) = \sqrt{3}x^{17} + 22.5x^{10} - \pi x^7 + \frac{1}{3}$
 Degree 17
 Leading term $\sqrt{3}x^{17}$
 Leading coefficient $\sqrt{3}$
 Constant term $\frac{1}{3}$
 As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$
 As $x \rightarrow \infty$, $f(x) \rightarrow \infty$
6. $s(t) = -4.9t^2 + v_0t + s_0$
 Degree 2
 Leading term $-4.9t^2$
 Leading coefficient -4.9
 Constant term s_0
 As $t \rightarrow -\infty$, $s(t) \rightarrow -\infty$
 As $t \rightarrow \infty$, $s(t) \rightarrow -\infty$
7. $P(x) = (x-1)(x-2)(x-3)(x-4)$
 Degree 4
 Leading term x^4
 Leading coefficient 1
 Constant term 24
 As $x \rightarrow -\infty$, $P(x) \rightarrow \infty$
 As $x \rightarrow \infty$, $P(x) \rightarrow \infty$
8. $p(t) = -t^2(3-5t)(t^2+t+4)$
 Degree 5
 Leading term $5t^5$
 Leading coefficient 5
 Constant term 0
 As $t \rightarrow -\infty$, $p(t) \rightarrow -\infty$
 As $t \rightarrow \infty$, $p(t) \rightarrow \infty$

9. $f(x) = -2x^3(x+1)(x+2)^2$

Degree 6

Leading term $-2x^6$ Leading coefficient -2

Constant term 0

As $x \rightarrow -\infty$, $f(x) \rightarrow -\infty$ As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$

10. $G(t) = 4(t-2)^2(t+\frac{1}{2})$

Degree 3

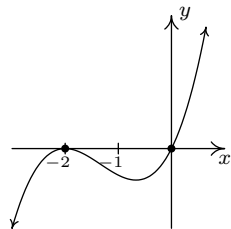
Leading term $4t^3$

Leading coefficient 4

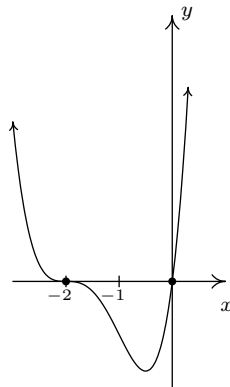
Constant term 8

As $t \rightarrow -\infty$, $G(t) \rightarrow -\infty$ As $t \rightarrow \infty$, $G(t) \rightarrow \infty$

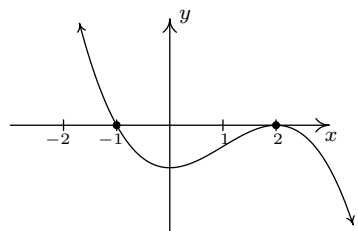
11. $a(x) = x(x+2)^2$

 $x = 0$ multiplicity 1 $x = -2$ multiplicity 2

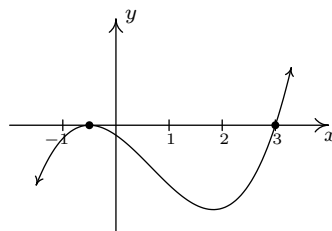
12. $g(x) = x(x+2)^3$

 $x = 0$ multiplicity 1 $x = -2$ multiplicity 3

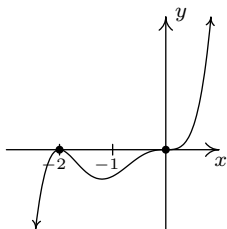
13. $f(x) = -2(x-2)^2(x+1)$

 $x = 2$ multiplicity 2 $x = -1$ multiplicity 1

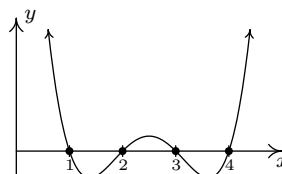
14. $g(x) = (2x+1)^2(x-3)$

 $x = -\frac{1}{2}$ multiplicity 2 $x = 3$ multiplicity 1

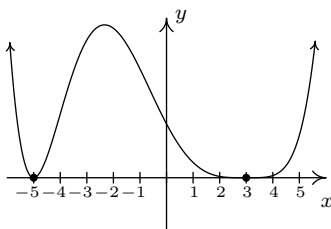
15. $F(x) = x^3(x+2)^2$
 $x = 0$ multiplicity 3
 $x = -2$ multiplicity 2



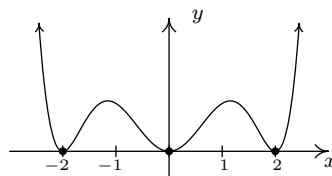
16. $P(x) = (x-1)(x-2)(x-3)(x-4)$
 $x = 1$ multiplicity 1
 $x = 2$ multiplicity 1
 $x = 3$ multiplicity 1
 $x = 4$ multiplicity 1



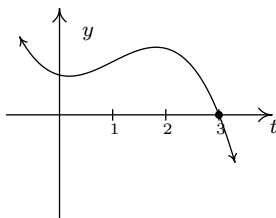
17. $Q(x) = (x+5)^2(x-3)^4$
 $x = -5$ multiplicity 2
 $x = 3$ multiplicity 4



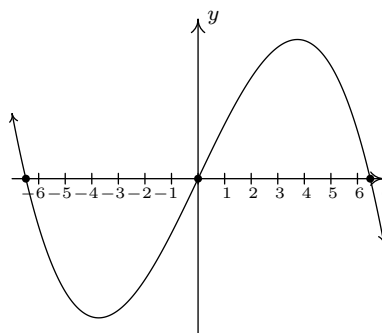
18. $f(x) = x^2(x-2)^2(x+2)^2$
 $x = -2$ multiplicity 2
 $x = 0$ multiplicity 2
 $x = 2$ multiplicity 2



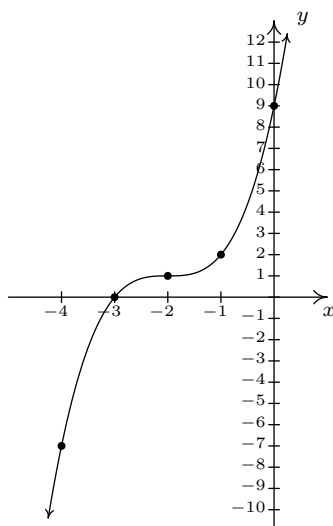
19. $H(t) = (3-t)(t^2+1)$
 $x = 3$ multiplicity 1



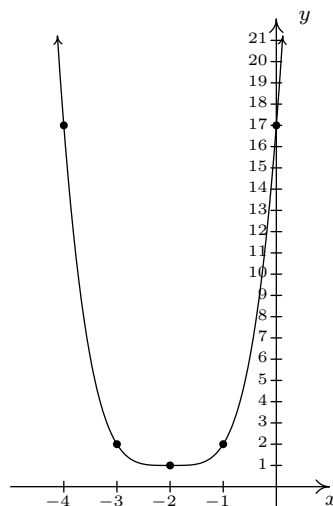
20. $Z(b) = b(42-b^2)$
 $b = -\sqrt{42}$ multiplicity 1
 $b = 0$ multiplicity 1
 $b = \sqrt{42}$ multiplicity 1



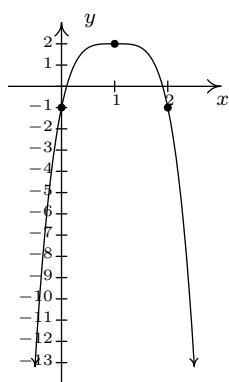
21. $g(x) = (x + 2)^3 + 1$
 domain: $(-\infty, \infty)$
 range: $(-\infty, \infty)$



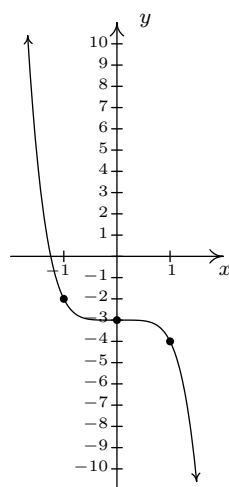
22. $g(x) = (x + 2)^4 + 1$
 domain: $(-\infty, \infty)$
 range: $[1, \infty)$



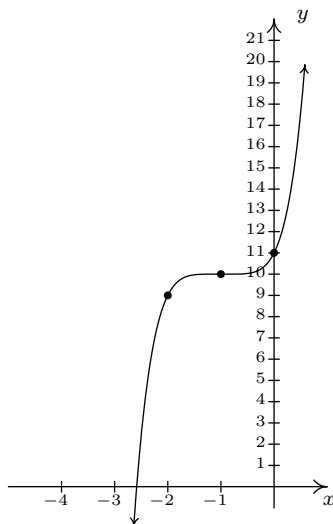
23. $g(x) = 2 - 3(x - 1)^4$
 domain: $(-\infty, \infty)$
 range: $(-\infty, 2]$



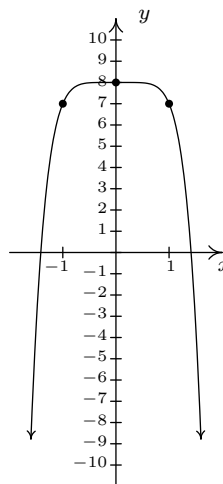
24. $g(x) = -x^5 - 3$
 domain: $(-\infty, \infty)$
 range: $(-\infty, \infty)$



25. $g(x) = (x + 1)^5 + 10$
 domain: $(-\infty, \infty)$
 range: $(-\infty, \infty)$



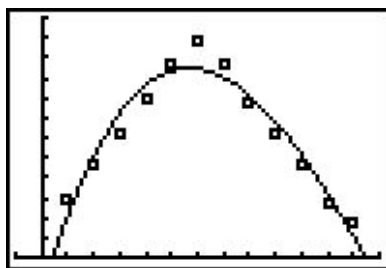
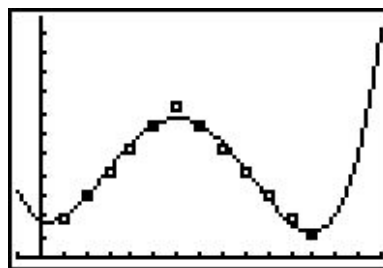
26. $g(x) = 8 - x^6$
 domain: $(-\infty, \infty)$
 range: $(-\infty, 8]$



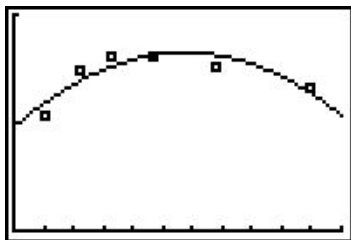
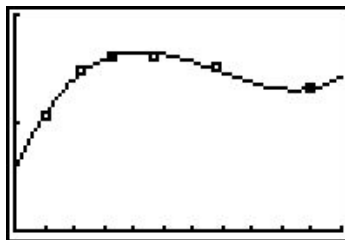
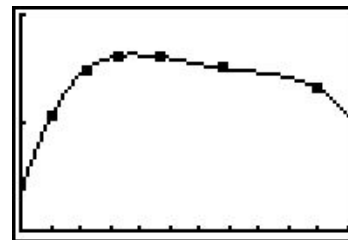
27. We have $f(-4) = -23$, $f(-3) = 5$, $f(0) = 5$, $f(1) = -3$, $f(2) = -5$ and $f(3) = 5$ so the Intermediate Value Theorem tells us that $f(x) = x^3 - 9x + 5$ has real zeros in the intervals $[-4, -3]$, $[0, 1]$ and $[2, 3]$.
28. $V(x) = x(8.5 - 2x)(11 - 2x) = 4x^3 - 39x^2 + 93.5x$, $0 < x < 4.25$. Volume is maximized when $x \approx 1.58$, so the dimensions of the box with maximum volume are: height ≈ 1.58 inches, width ≈ 5.34 inches, and depth ≈ 7.84 inches. The maximum volume is ≈ 66.15 cubic inches.
29. The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 6.305$ and $y \approx 1115.417$. Since x represents the number of TVs sold in hundreds, $x = 6.305$ corresponds to 630.5 TVs. Since we can't sell half of a TV, we compare $R(6.30) \approx 1115.415$ and $R(6.31) \approx 1115.416$, so selling 631 TVs results in a (slightly) higher revenue. Since y represents the revenue in *thousands* of dollars, the maximum revenue is \$1,115,416.
30. $P(x) = R(x) - C(x) = -5x^3 + 35x^2 - 45x - 25$, $0 \leq x \leq 10.07$.
31. The calculator gives the location of the absolute maximum (rounded to three decimal places) as $x \approx 3.897$ and $y \approx 35.255$. Since x represents the number of TVs sold in hundreds, $x = 3.897$ corresponds to 389.7 TVs. Since we can't sell 0.7 of a TV, we compare $P(3.89) \approx 35.254$ and $P(3.90) \approx 35.255$, so selling 390 TVs results in a (slightly) higher revenue. Since y represents the revenue in *thousands* of dollars, the maximum revenue is \$35,255.
32. Making and selling 71 PortaBoys yields a maximized profit of \$5910.67.

33. (a) Our ultimate goal is to maximize the volume, so we'll start with the maximum Length + Girth of 130. This means the length is $130 - 4x$. The volume of a rectangular box is always length \times width \times height so we get $V(x) = x^2(130 - 4x) = -4x^3 + 130x^2$.
- (b) Graphing $y = V(x)$ on $[0, 33] \times [0, 21000]$ shows a maximum at $(21.67, 20342.59)$ so the dimensions of the box with maximum volume are 21.67in. \times 21.67in. \times 43.32in. for a volume of 20342.59in.³.
- (c) If we start with Length + Girth = 108 then the length is $108 - 4x$ and the volume is $V(x) = -4x^3 + 108x^2$. Graphing $y = V(x)$ on $[0, 27] \times [0, 11700]$ shows a maximum at $(18.00, 11664.00)$ so the dimensions of the box with maximum volume are 18.00in. \times 18.00in. \times 36in. for a volume of 11664.00in.³. (Calculus will confirm that the measurements which maximize the volume are exactly 18in. by 18in. by 36in., however, as I'm sure you are aware by now, we treat all calculator results as approximations and list them as such.)
34. The cubic regression model is $p_3(x) = 0.0226x^3 - 0.9508x^2 + 8.615x - 3.446$. It has $R^2 = 0.93765$ which isn't bad. The graph of $y = p_3(x)$ in the viewing window $[-1, 13] \times [0, 24]$ along with the scatter plot is shown below on the left. Notice that p_3 hits the x -axis at about $x = 12.45$ making this a bad model for future predictions. To use the model to approximate the number of hours of sunlight on your birthday, you'll have to figure out what decimal value of x is close enough to your birthday and then plug it into the model. My (Jeff's) birthday is July 31 which is 10 days after July 21 ($x = 7$). Assuming 30 days in a month, I think $x = 7.33$ should work for my birthday and $p_3(7.33) \approx 17.5$. The website says there will be about 18.25 hours of daylight that day. To have 14 hours of darkness we need 10 hours of daylight. We see that $p_3(1.96) \approx 10$ and $p_3(10.05) \approx 10$ so it seems reasonable to say that we'll have at least 14 hours of darkness from December 21, 2008 ($x = 0$) to February 21, 2009 ($x = 2$) and then again from October 21, 2009 ($x = 10$) to December 21, 2009 ($x = 12$).

The quartic regression model is $p_4(x) = 0.0144x^4 - 0.3507x^3 + 2.259x^2 - 1.571x + 5.513$. It has $R^2 = 0.98594$ which is good. The graph of $y = p_4(x)$ in the viewing window $[-1, 15] \times [0, 35]$ along with the scatter plot is shown below on the right. Notice that $p_4(15)$ is above 24 making this a bad model as well for future predictions. However, $p_4(7.33) \approx 18.71$ making it much better at predicting the hours of daylight on July 31 (my birthday). This model says we'll have at least 14 hours of darkness from December 21, 2008 ($x = 0$) to about March 1, 2009 ($x = 2.30$) and then again from October 10, 2009 ($x = 9.667$) to December 21, 2009 ($x = 12$).


 $y = p_3(x)$

 $y = p_4(x)$

35. (a) The scatter plot is shown below with each of the three regression models.
- (b) The quadratic model is $P_2(x) = -0.02x^2 + 0.241x + 0.956$ with $R^2 = 0.77708$.
 The cubic model is $P_3(x) = 0.005x^3 - 0.103x^2 + 0.602x + 0.573$ with $R^2 = 0.98153$.
 The quartic model is $P_4(x) = -0.000969x^4 + 0.0253x^3 - 0.240x^2 + 0.944x + 0.330$ with $R^2 = 0.99929$.
- (c) The maximums predicted by the three models are $P_2(5.737) \approx 1.648$, $P_3(4.232) \approx 1.657$ and $P_4(3.784) \approx 1.630$, respectively.


 $y = P_2(x)$

 $y = P_3(x)$

 $y = P_4(x)$